

# Performance comparison of Rate $\frac{1}{2}$ Convolutional Codes with BPSK on Rayleigh and AWGN channels for Memory or Memory less condition

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**Abstract**—This paper is concerned with the  $\frac{1}{2}$  Binary convolutional codes (BCC) in conjunction with Binary Phase Shift Keying (BPSK) modulation and maximum likelihood Viterbi decoding on AWGN and Rayleigh channel. Applying the Viterbi algorithm with hard decision decoding comparison of Bit Error Rate versus Signal to Noise ratio in AWGN and Rayleigh channel is shown for memory or memory less condition.

**Keywords**—BPSK, Convolutional Encoder, Viterbi Decoder, Rayleigh fading channel.

## I. INTRODUCTION

Digital Communications over mobile channels often suffer from multipath effects, which result in signal fading. It is known that fading degrades the performance of communication system. To combat fading Viterbi algorithm could be used. Viterbi decoders are used to decode convolutional coding, which has been used in deep Space communications as well as wireless communications. A wireless cellular standard for CDMA (code division multiple access), IS-95 employs convolutional coding. As convolutional codes become more powerful the complexity of corresponding decoders generally increases. The Viterbi algorithm, which is the most extensively employed decoding algorithm for convolutional codes, works well for less-complex codes, indicated by constraint length  $K$ .

The viterbi algorithm was proposed in 1967 as a method of decoding convolution codes [11]. However, the algorithm's memory requirement and computation speed pose a performance obstacle when decoding more powerful codes with large constraint lengths. In order to overcome this problem, the adaptive Viterbi algorithm (AVA) has been developed. The decoder performs joint successive interference cancellation (SIC) and Viterbi decoding (VD) in each step of the interference cancellation [10].

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This algorithm reduced the average number of computations required per bit of decoded information while achieving comparable bit error rates (BER). Reconfigurable computing has been proposed for signal processing with various objectives, including high performance, flexibility, specialization, and most recently, adaptability. Reconfiguration is characterized by how fast the reconfiguration can occur and how many possible reconfigurations can be used. For many signal-processing systems, it is possible to exploit variations in signals to vary computation and memory requirements. The decoding of convolutional codes with the viterbi algorithm has been used very successful on satellite and space channels where the channel is memoryless. And viterbi algorithm is also successful in those channel have memory.[9]

The recent proliferations of wireless communication systems have indicated the need of low BER communications architectures at the hardware level. These architectures can be characterized with a set of architectural parameters, which can be determined experimentally. Chan and Lee implemented an adaptive Viterbi decoder for high-speed communication. The adaptive decoder discards some states (in the trellis) with high path metrics dynamically during the decoding process. Seki et al and Lang et al suggested the use of a scarce state transition (SST) scheme. The scheme employs a simple pre-decoder followed by a pre-encoder to minimize signal transitions at the input of a conventional Viterbi decoder, which leads to lower bit error rate (BER). Kang and Wilson studied various issues in designing a low probability of error (Pe) Viterbi decoder for the IS-95 CDMA system.

Additive white Gaussian noise (AWGN) is a channel model in which the only impairment to communication is a linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for fading, frequency selectivity, interference, nonlinearity or dispersion. However, it produces simple and tractable mathematical models which are useful for gaining insight into the underlying behavior of a system before these other phenomena are considered.

Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices [12].

Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communications channel) will vary randomly, or fade, according to a Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables. Rayleigh fading is viewed as a reasonable model for tropospheric and ionospheric signal propagation as well as the effect of heavily built-up urban environments on radio signals. In this paper we have compared the BER versus SNR for BCC with Viterbi decoding for BPSK modulation scheme with hard decoding for finite survivor state memory and memory less.

## II. CHANNEL MODEL

### A. Additive White Gaussian Noise(AWGN) Channel

In communications, the AWGN channel model is one in which the only impairment is the linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for the phenomena of fading, frequency selectivity, interference, nonlinearity or dispersion. However, it produces simple, tractable mathematical models which are useful for gaining insight into the underlying behavior of a system before these other phenomena are considered. AWGN is commonly used to simulate background noise of the channel under study, in addition to multipath, terrain blocking, interference, ground clutter and selfinterference that modern radio systems encounter in terrestrial operation.

### B. Rayleigh Fading Channel

In the flat Rayleigh channel model, the channel is assumed to introduce two random variables in the signal observed by the receiver; a random fluctuation of the received signal energy and phase and an additive, white Gaussian noise component, as seen in the system block diagram of Fig 1. Assuming that the receiver is able to perfectly track the phase of the channel, the detector in the receiver observes the signal  $r = |a|s+n$ , where  $a$  is the complex channel coefficient. The random variable  $|a|$  has a Rayleigh probability density function with mean  $\pi/2$  and  $n = nI + jnQ$  is a complex Gaussian noise variable with  $nI$  and  $nQ$  being identically distributed zero-mean Gaussian random variables with variance  $N0/2$ , where  $N0/2$  is the double-sided noise power spectral density. Each signal alternative is associated with a decision region  $Sk$ , and the receiver determines what  $Sk$  the received signal  $r$  falls within and outputs as its estimate  $\hat{s}$  the corresponding signal  $sk$ . A bit error is said to occur if the estimate  $\hat{s}$  differs from the transmitted signal  $s$ .

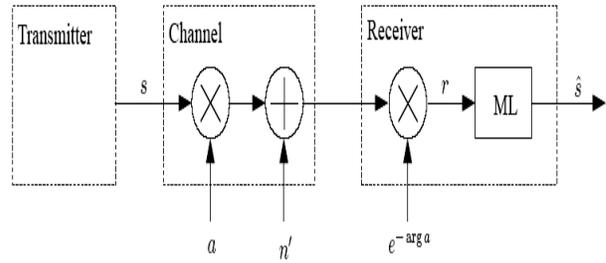


Fig. 1. Schematic block diagram of a communication system communicating over a Rayleigh fading channel.

## III. CONVOLUTIONAL CODE

A convolutional code is a type of error-correcting code in which (a) each m-bit information symbol (each m-bit string) to be encoded is transformed into an n-bit symbol, where m/n is the code rate ( $n \geq m$ ) and (b) the transformation is a function of the last k information symbols, where k is the constraint length of the code. Fig. 2 shows a constraint length of 5, code rate 1/2 convolutional encoder.

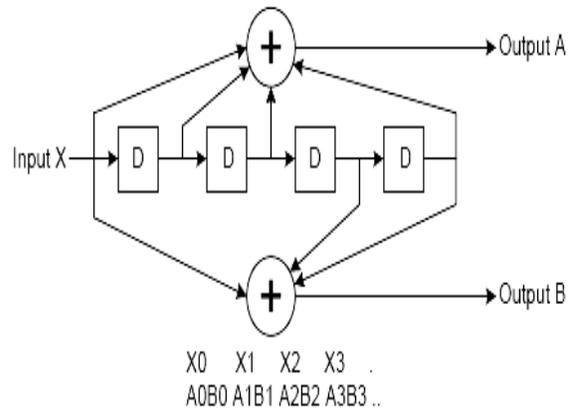


Fig. 2. Convolutional encoder.

## IV. VITERBI DECODER ALGORITHM

The VDA is widely used today for various applications in digital communication, of which the decoding of convolutional codes is probably most important. The trellis of convolutional codes constructed by drawing the possible state transitions of the encoder which is a feed forward shift register with parallel output over time. State of shift register, any sequence of input bits corresponds to unique path through trellis. A trellis (Fig.3) is described by the finite number of N states,  $z_i$  ( $i \in \{0, 1, \dots, N-1\}$ ) of the encoder shift registers at every discrete time instant k and by branches with associated channel symbols representing the state transition of the time intervals  $(k, k+1)$  that connect states.

Using the observed channel symbols  $c_i$ , a weight called transition metric  $\lambda_{ij}$ , k is derived for every possible state transition from state  $z_j$  to state  $z_i$  as a measure of probability

for the corresponding state transition. These transition metrics are accumulated as path metric for the paths given by successive transitions. Fig. 3 shows the Viterbi algorithm is essentially a shortest path algorithm for computing the shortest path through the trellis associated with the code.

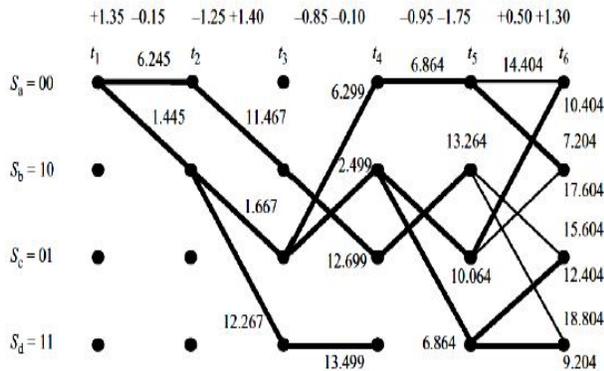


Fig. 3. Trellis diagram of the Viterbi decoder

### V. HARD DECISION DECODING

Hard decision decoding takes a stream of bits say from the 'threshold detector' stage of a receiver, where each bit is considered definitely one or zero. Eg. for binary signaling, received pulses are sampled and the resulting voltages are compared with a single threshold. If a voltage is greater than the threshold it is considered to be definitely a 'one' say regardless of how close it is to the threshold. If it is less, it is definitely zero.

The post-photo detector noise in optical fiber communication systems, however, has non-Gaussian statistics. It would be time and cost efficient to predict FEC performance in optical fiber channels based on the existing results in AWGN channels without redoing all the evaluations with a more complex non-Gaussian channel model.

The Viterbi algorithm finds the closet coded sequence  $v$  to the received sequence  $r$  by processing the sequences on an information bit-by-bit (branches of the trellis) basis. Instead of keeping a score of each possible coded sequence, the Viterbi decoder tracks the states of the trellis. The likelihood of a received sequence  $r$  after transmission over a noisy channel, given that a coded sequence  $v$  is sent, is given by the conditional probability:

$$P(\bar{r}|\bar{v}) = \prod_{i=0}^{n-1} P(r_i|v_i)$$

For the BSC channel, a maximum-likelihood decoder (MLD) is equivalent to choosing the code sequence that minimizes the Hamming distance (hard decoding):

$$d_H(\bar{r}, \bar{v}) = \sum_{i=0}^{n-1} d_H(r_i, v_i)$$

### VI. TRACEBACK METHOD

In a Viterbi Decoder, there are two known memory organization techniques for the storage of survivor sequences from which the decoded information sequence is retrieved. The register exchange method is the simplest conceptually but suffers from the disadvantage that every bit in the memory must be read and rewritten for each information bit decoded. The alternative is the traceback method where the interpretation of the symbols as pointers removes the necessity to move data in the memory.

The Traceback method stores path information in the form of an array of recursive pointers. Unfortunately, direct implementation of the Traceback method proposed is impossible, since it treats memory as infinite in size, while any actual implementation contains only a finite memory. It is advantageous to think of traceback memory as organized in a two-dimensional structure, with rows and columns.

The number of rows is equal to the number of states  $N = 2^v$ . Each column stores the results of  $N$  comparisons corresponding to a Trellis stage-Time. Since the stream of symbols is in general, semi-infinite, storage locations are periodically reused. There are three types of operations performed inside a TB decoder:

*Traceback read (tb)* - This is one of the two read operations and consists of reading a bit and interpreting this bit in conjunction with the present state number as a pointer that indicates the previous state number (i.e. state number of the predecessor). Pointer values from this operation are not output as decoded values, instead they are used to ensure that all paths have converged with some high probability, so that actual decoding may take place. The traceback operation is usually run to a predetermined depth,  $T$ , before being used to initiate the decode read operation.

*Decode read (dc)* - This operation proceeds in exactly the same fashion as the traceback operation, but operates on older data, with the state number of the first decode read in a memory bank being determined by the previously completed traceback. Pointer values from this operation are the decoded values and are sent to the bit-order reversing circuit. A decode read can serve as a dual decode and traceback read, this allows us to decode read multiple columns using one traceback read operation of  $T$  columns.

*Writing new data (wr)* - The decisions made by the ACS are written into locations corresponding to the states. The write pointer advances forward as ACS operations move from one stage to the next in the trellis, and data are written to locations just freed by the decode read operation. For every set of column write operations ( $N$  bits wide), an average of one decode read must be performed. The overhead of  $T$ -column traceback read can be spread over one or more column decode read operations, resulting in  $k > 1$  read operations, this includes both decode read operations and traceback read operations.

The One-Pointer Algorithm is the best amongst the known TB methods. It requires approximately half as much memory as either the of k-Pointer Even or Odd Algorithms. The latency is similarly reduced by a factor of two. In addition, the number of memory modules required is also half as large as that required by the k-Pointer Algorithms. The only disadvantage of the One-Pointer Algorithm is the need to provide separate column counters for the write operations and for the read operations, since the read counter advances by k columns for every one column advance of the write pointer. If k is selected to be a power of two, say  $2^b$ , then the read counter can be implemented simply by using the b most significant bits of the write row counter as b least significant bits of the read counter.

In a fully parallel system (i.e. all  $2^v$  decisions of a given stage are computed and written simultaneously on the same clock cycle), the k-Pointer Even Algorithm, with only a single read operation per pointer per clock cycle, is better than the One-Pointer Algorithm that requires k read operations per pointer per clock cycle.

In order to significantly boost the throughput of Viterbi Decoders, researchers are increasingly turning to use of multiple processing units. Although full details cannot be given in this short paper, we must note that the advantages of the TB method over the RE method become even more pronounced when Survivor Sequence memory is distributed. Selection of the proper value of IC allows one to trade off the latency against the number of memory banks (i.e. complexity of controls).

Traceback with finite survivor memory: To be able to decode with finite survivor memory, let's say we need to start the traceback at some time instant  $D+TB$ , where D is the decoding depth and TB is the traceback depth.

The sequences of events are as follows:

- At time instant  $D+TB$ , we start the traceback and continue tracing back through the survivor path TB times. Once we are done with the traceback, we start estimating the D decoded bits knowing the current state and previous state.
- Similarly, we again start the traceback at time  $2D+TB$ , do traceback TB times and then decode bits from  $2D$  to  $D$ .
- Once we reach the end of the input sequence at time instance  $N+K-1$ , we know that trellis has converged to state00 and then we do demodulation with traceback.

**1) Selecting start state for traceback:**

At time instant  $D+TB$ , there are multiple ways to select the state to

- Always start from a defined state (for example state 00)
- Start from the state which has the minimum path metric

From simulations, we can identify the minimum value of traceback depth (TB), which results in performance comparable to infinite survivor path memory case.

From simulations it has been observed that doing traceback depth of around 15 ensures that the obtained BER with finite survivor state memory is close to the BER with infinite survivor state memory. Note that  $TB=15$  corresponds to around 5 times the constraint length of  $K=3$ .

Additionally, as expected starting traceback with from the

state with minimum pathmetric shows better performance than always starting at state 00.

**VII. PERFORMANCE COMPARISON**

We have observed that applying the techniques of traceback method in Viterbi algorithm when we use hard decision the BER vs SNR curve improve much than the normal un coded signal for both Rayleigh and AWGN channels with memory or memory less. In Fig. 5 and Fig.6 we have shown BER improvement in AWGN channel by using traceback and without traceback method with viterbi algorithm about 1.5 dB. Fig. 7 shows BER improvement in flat Rayleigh fading channel without traceback unit by viterbi algorithm about 15 dB but by using traceback unit in flat Rayleigh fading channel we get almost 17 dB improvement. In Fig. 4 and Fig.9 we have shown a comparison of BER between AWGN channel and flat Rayleigh fading channel with traceback and without traceback method respectively.

Combine BER for BCC with Viterbi decoding (TB=15) for BPSK in Rayleigh fading and AWGN channel

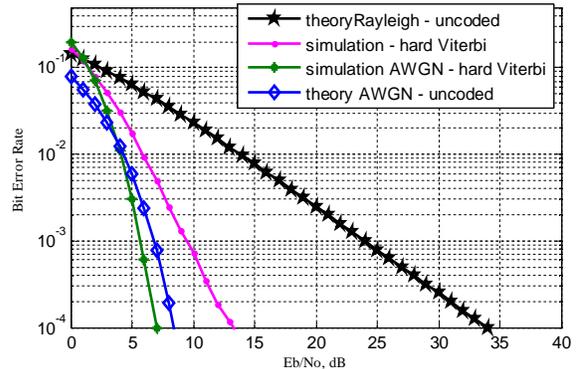


Fig. 4. BER vs  $E_b/N_0$  for un coded Rayleigh, AWGN, Hard viterbi and BCC Hard viterbi with AWGN channel with memory

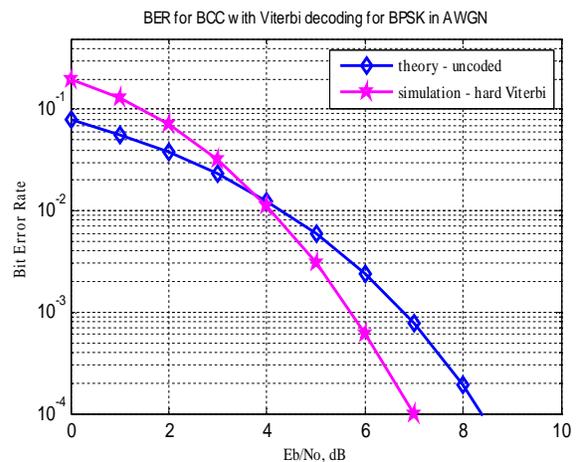


Fig. 5. BER vs  $E_b/N_0$  for un coded AWGN and BCC Hard viterbi with AWGN memory less channel

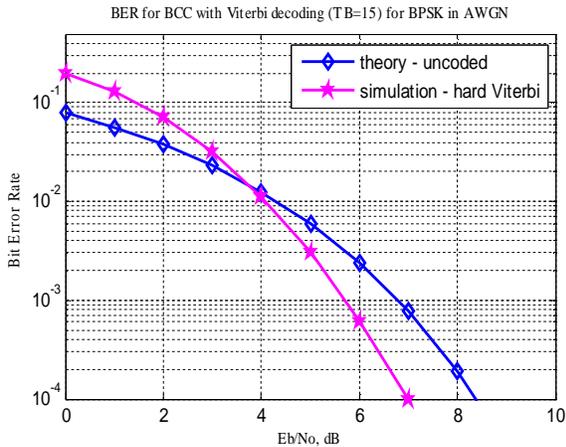


Fig. 6. BER vs  $E_b/N_0$  for un coded AWGN and BCC Hard viterbi with AWGN with memory channel

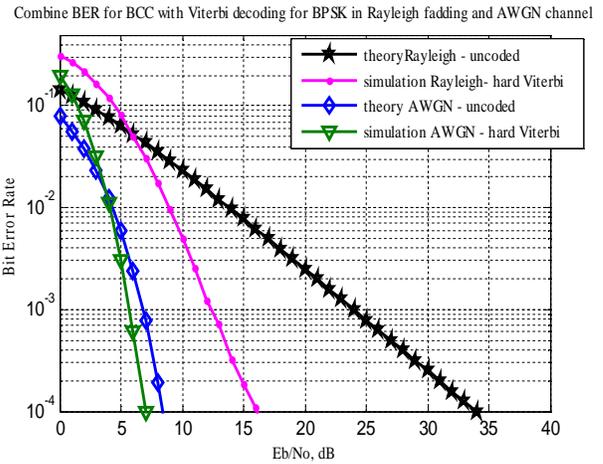


Fig. 9. BER vs  $E_b/N_0$  for un coded Rayleigh, AWGN, Hard viterbi and BCC Hard viterbi with AWGN channel with memory less

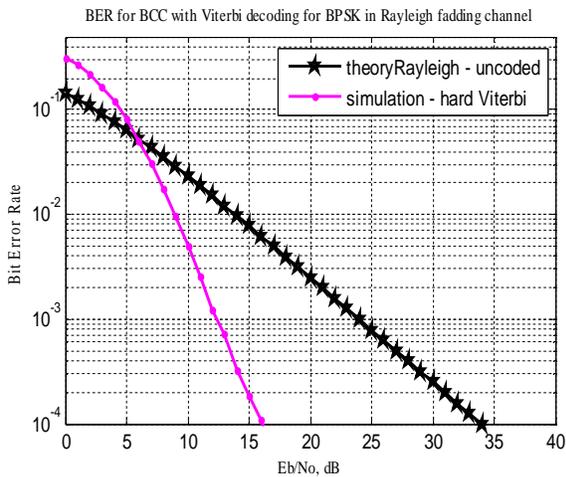


Fig. 7. BER vs  $E_b/N_0$  for un coded Rayleigh and BCC Hard viterbi with AWGN with memory less channel

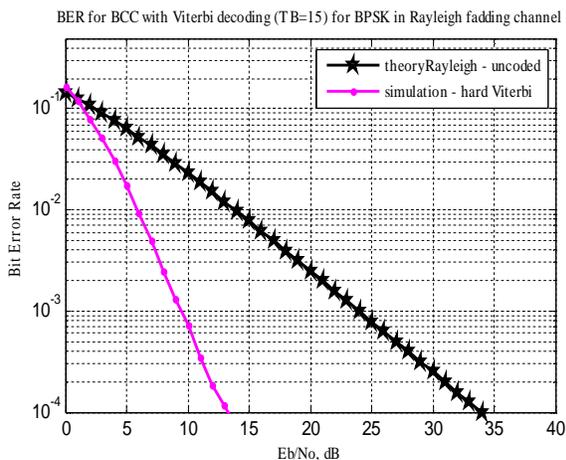


Fig. 8. BER vs  $E_b/N_0$  for un coded Rayleigh and BCC Hard viterbi with AWGN with memory channel

### VIII. CONCLUSION

This paper has presented traceback memory management in Viterbi Decoders. For a high speed, large constraint length VD the Traceback algorithm is advantageous as compared to the Register Exchange method. TB method is superior because of the lower bandwidth requirements (lower power dissipation). The TB method is particularly suitable for use in a multiprocessor implementation of the VD with memory distributed among the processors; in a multiprocessor VD the TB method requires one to two orders of magnitude less area than the RE method.

This paper also shows the performance comparison when the hard decision is used in Viterbi Algorithm when it is associated with memory or not, which can be very helpful for decision making for channel choosing.

### REFERENCE

- [1] C. M. Rader. Memory Management in a Viterbi Algorithm. IEEE "transactions on Communications, 29:1399-1401, September 1981.
- [2] G. Feygin, P. G. Gulak, and F. Pollara. Survivor Sequence Memory Management in Viterbi Decoders. In Third IEM Workshop on ECC, San Jose, California, September 1989
- [3] A. J. Viterbi, "Convolutional codes and their performance in communications", IEEE Trans. Commun. Technol., vol. com-19, pp.751-771, Oct 1971.
- [4] R. M. Orndorf et al., Viterbi Decoder VLSI Integrated circuit for Bit Error Correction, Anaheim, CA: Rockwell International, Dec. 1981.
- [5] G. D. Forney Jr. The Viterbi algorithm. Proc. IEEE- 61:268-278, March, 1973.
- [6] Chip Fleming, 'A tutorial on convolutional coding with Viterbi algorithm', 2003.
- [7] John G. Proakis, 'Digital Communications', Mc. Graw Hill, 4th edition, 2001.
- [8] Simon Haykin, 'Communication Systems', 4th edition, John Wiley & sons, Inc. 2001.
- [9] J. Hagenauer "Viterbi decoding of Convolutional codes for fading and burst channels" IEEE seminar on Digital communications, Catalog No.80CH1521-4,pp G2.1- 7,Zurich1980. Y. Sanada and Q. Wang, "A Co-Channel Interference
- [10] Cancellation Technique Using Orthogonal Convolutional Codes," IEEE Trans. on Communications," Vol.44, No. 5, May 1996, pp. 549-556.
- [11] Andrew J. Viterbi, "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm", IEEE Transactions on Information Theory, Volume IT-13, pages 260-269, in April, 1967.

- [12] Bernard Sklar, "Rayleigh Fading Channel in Mobile Digital Communication System Part I: Characterization", IEEE Communication Magazine, pp. 90-100, July 1997.



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