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Applications of Graph Coloring in Modern Computer Science

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***Abstract*—Graph coloring is one of the most important concepts in graph theory and is used in many real time applications in computer science. The main aim of this paper is to present the importance of graph coloring ideas in various areas of compute applications for researches that they can use graph coloring concepts for the research. Graph coloring used in various research areas of computer science such data mining, image segmentation, clustering, image capturing, networking etc. This papers mainly focused on important applications such as Guarding an Art Gallery, Physical layout segmentation, Round-Robin Sports Scheduling, Aircraft scheduling, Biprocessor tasks, Frequency assignment, Final Exam Timetabling as a Grouping Problem, Map coloring and GSM mobile phone networks, and Student Time Table.**

***Index Terms*—Graph Theory, Graph Coloring, Guarding an Art Gallery, Physical Layout Segmentation, Map Coloring, Timetabling and Grouping Problems, Scheduling Problems, Graph Coloring Applications.**

I. INTRODUCTION

The origin of graph theory started with the problem of Koinsber bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles [1]) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R.Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H.Dudeney mentioned a puzzle problem. Eventhough the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory.

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Graph theoretical concepts are widely used to study and model various applications, in different areas. They include, study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actor prestige or to explore diffusion mechanisms. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affects other species. Graph theoretical ideas are highly utilized by computer science applications.

Graph coloring especially used various in research areas of computer science such data mining, image segmentation, clustering, image capturing, networking etc., For example a data structure can be designed in the form of tree which in turn utilized vertices and edges. Similarly modeling of network topologies can be done using graph concepts. In the same way the most important concept of graph coloring is utilized in resource allocation, scheduling. Also, paths, walks and circuits in graph theory are used in tremendous applications say traveling salesman problem, database design concepts, resource networking. This leads to the development of new algorithms and new theorems that can be used in tremendous applications.

Graph coloring is one of the most important concepts in graph theory and is used in many real time applications in computer science. Various coloring methods are available and can be used on requirement basis. The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph [1].

II. APPLICATIONS OF GRAPH COLORING

A. *Guarding an Art Gallery*

The application of Graph Coloring also used in guarding an art gallery. Art galleries therefore have to guard their collections carefully. During the day the attendants can keep a look-out, but at night this has to be done by video cameras. These cameras are usually hung from the ceiling and they rotate about a vertical axis. The images from the cameras are sent to TV screens in the office of the night watch. Because it is easier to keep an eye on few TV screens rather than on many, the number of cameras should be as small as possible. An additional advantage of a small number of cameras is that the cost of the security system will be lower. On the other hand we cannot have too few cameras, because every part of the gallery must be visible to at least one of them. So we should place the cameras at strategic positions, such that each of them guards a large part of the gallery.

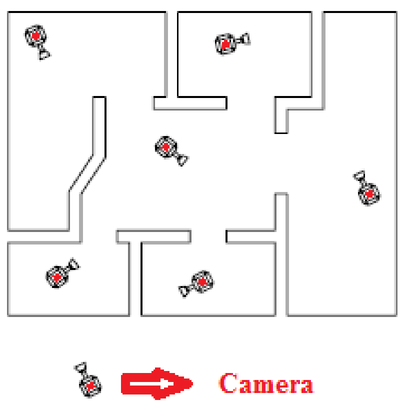

Fig 1. Cameras are used for guarding an art gallery.

If we want to define the art gallery problem more precisely, we should first formalize the notion of gallery. A gallery is, of course, a 3-dimensional space, but a floor plan gives us enough information to place the cameras. Therefore we model a gallery as a polygonal region in the plane. We further restrict ourselves to regions that are simple polygons, that is, regions enclosed by a single closed polygonal chain that does not intersect itself. Thus we do not allow regions with holes. A camera position in the gallery corresponds to a point in the polygon. A camera sees those points in the polygon to which it can be connected with an open segment that lies in the interior of the polygon.

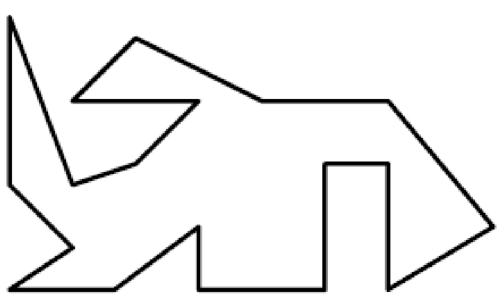

Fig 2. A Simple Polygon.

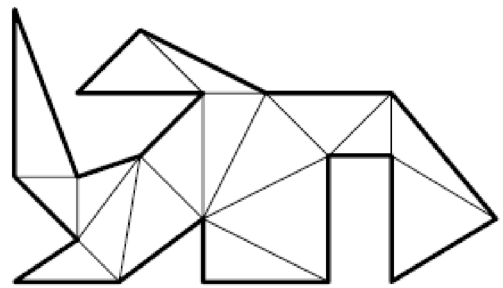

Fig 3. A Possible Triangulation of the Polygon in Fig 2.

A triangulated simple polygon can always be 3-colored. As a result, any simple polygon can be guarded with $\lfloor n/3 \rfloor$ cameras. But perhaps we can do even better. After all, a camera placed at a vertex may guard more than just the incident triangles. Unfortunately, for any n there are simple polygons that require $\lfloor n/3 \rfloor$ cameras. For a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are occasionally necessary and always sufficient to have every point in the polygon visible from at least one of the cameras.

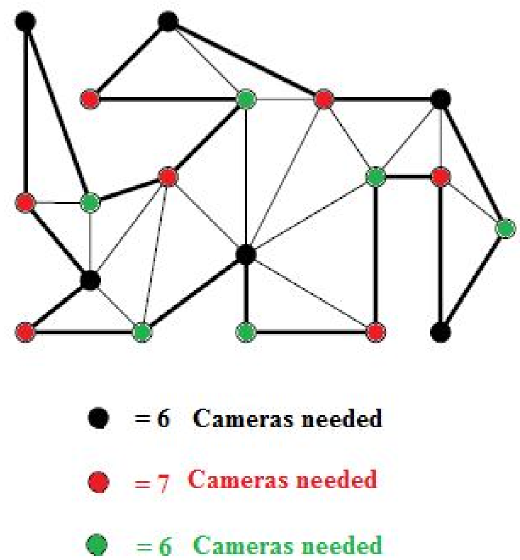

Fig 4. A 3-Coloring of the Triangulated Polygon in Fig 3, take the smallest color class to place the cameras. Here, we can choose black or green color class to place the cameras.

Now we know that $\lfloor n/3 \rfloor$ cameras are always sufficient. But we don't have an efficient algorithm to compute the camera positions yet. What we need is a fast algorithm for triangulating a simple polygon. The algorithm should deliver a suitable representation of the triangulation a doubly-connected edge list, for instance so that we can step in constant time from a triangle to its neighbors. Given such a representation, we can compute a set of at most $\lfloor n/3 \rfloor$ camera positions in linear time with the method described above: use depth first search on the dual graph to compute a 3-colouring and take the smallest color class to place the cameras. In the coming sections we describe how to compute a triangulation in O(nlogn) time. Anticipating this, we already state the final result about guarding a polygon.

B. Physical Layout Segmentation

Automatic mail sorting machines of most recent systems process about 17 mail pieces per second. That requires a fast and precise OCR based recognition of the block-address. This recognition is mainly conditioned by a correct address lines organization [2][3][4]. Once the envelope image has been acquired by a linear CCD camera, three principal modules contribute to the task of the address-block localization: physical layout segmentation of envelope image, feature extraction and address-block identification.

Every-day, the postal sorting systems diffuse several tons of mails. It is noted that the principal origin of mail rejection is related to the failure of address-block localization task, particularly, of the physical layout segmentation stage. The bottom-up and top-down segmentation methods bring different knowledge that should not be ignored when we need to increase the robustness. Hybrid methods combine the two strategies in order to take advantages of one strategy to the detriment of other. Starting from these remarks, our proposal makes use of a hybrid segmentation strategy more adapted to the postal mails. The high level stages are based on the hierarchical graphs coloring. Today, no other work in this context has make use of the powerfulness of this tool. The performance evaluation of our approach was tested on a corpus of 10000 envelope images. The processing times and the rejection rate were considerably reduced.

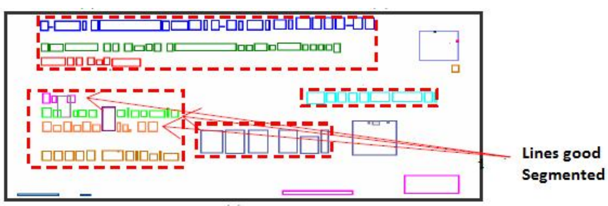

Fig. 5. Perception of the text lines by hierarchical graph coloring.

The segmentation technique objective is based on its decision strategy which defines a best block extraction manner in order to recognize it by the block address recognition module. The segmentation techniques cannot systematically produce uniform and good located blocks in complex environments (difficult envelopes). Consequently, the knowledge delivered by the descriptors of non-homogeneous blocks (containing parasitic elements) is less discriminating. In order to improve the robustness and exactness of segmentation, it has been necessary to choose an even more advanced tool. The idea is to use a hybrid strategy of segmentation using the richness of pyramidal structure. Our method is mainly based on the powerfulness of graph coloring to regroup correctly the connected components into text lines then the lines into blocks.

C. Round-Robin Spots Scheduling

Round-robin (RR) sports schedules occur in many tournaments and leagues across the globe, including the Six Nations Rugby Championships, various European and South American domestic soccer leagues, and the England and Wales County Cricket Championships. Round-robins are schedules involving n teams, where each team is required to play all other teams exactly m times within a fixed number of rounds. The most common types are single round-robins (SRRs), where m = 1, and double round- robins (DRRs), where m = 2 (in the latter, teams will typically be scheduled to meet once in each other's home venue).

The issue of producing valid, compact round-robin sports schedules by considering the problem as one of graph coloring. Using this model, which can also be extended to incorporate additional constraints, the difficulty of such problems is then gauged by considering the performance of a number of different graph coloring algorithms. In many cases solutions to round-robin scheduling problems can be produced via the utilization of well-known graph coloring principals. In particular, we have seen that three existing, heuristic-based graph coloring methods have been able to construct valid compact round-robin schedules, often in the presence of a large number of additional hard constraints. However, we have also noted areas in which some of these methods seem to struggle, such as when tackling very large instances, or those in the noted phase transition regions.

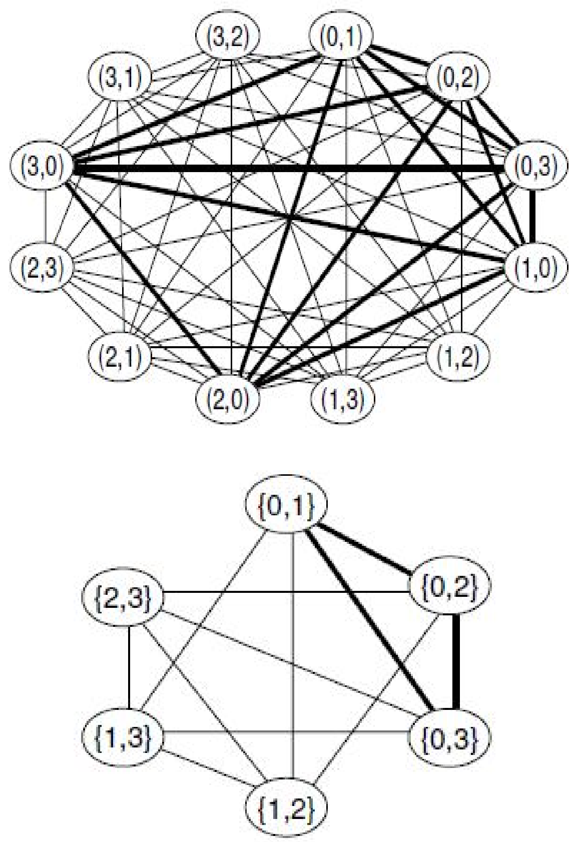

Fig. 6. Graphs for single (left) and double (right) round-robin problems with n = 4.

D. Aircraft Scheduling

Assume that we have k aircrafts, and we have to assign them to n flights, where the ith flight is during the time interval (ai, bi). Clearly, if two flights overlap, then we cannot assign the same aircraft to both flights. The vertices of the conflict graph correspond to the flights, two vertices are connected if the corresponding time intervals overlap. Therefore the conflict graph is an interval graph, which can be colored optimally in polynomial time.

E. Biprocessor Tasks

Assume that we have a set of processors (machines) and a set of tasks, each task has to be executed on two preassigned processors simultaneously. A processor cannot work on two jobs at the same time. For example, such biprocessor tasks arise when we want to schedule file transfers between processors [5] or in the case of mutual diagnostic testing of processors [6]. Consider the graph whose vertices correspond to the processors, and if there is a task that has to be executed on processors i and j, then we add an edge between the two corresponding vertices. Now the scheduling problem can be modeled as an edge coloring of this graph: we have to assign colors to the edges in such a way that every color appears at most once at a vertex. Edge coloring is **NP**-hard [7], but there are good approximation algorithms. The maximum degree Δ of the graph is an obvious lower bound on the number of colors needed to color the edges of the graph. On the other hand, if there are no multiple edges in the graph (there are no two tasks that require the same two processors), then Vizing's Theorem gives an efficient method for obtaining a $(\Delta + 1)$-edge coloring. If multiple edges are allowed, then the algorithm of [8] gives an 1:1-approximate solution.

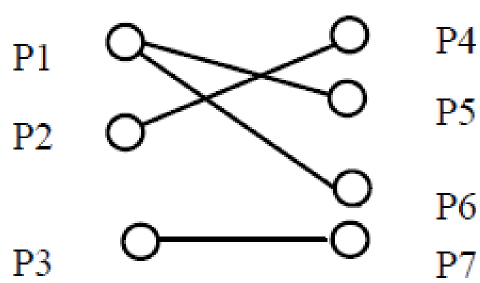

Fig. 7. Tasks allocated to processors, the diagram shows the tasks namely task1, task2, task3 and task4 are allocated to the processors (P1, P5); (P1, P6); (P2, P4) and (P3, P7) respectively

F. Frequency Assignment

Assume that we have a number of radio stations, identified by x- and y- coordinates in the plane. We have to assign a frequency to each station, but due to interferences, stations that are "close" to each other have to receive different frequencies. Such problems arise in frequency assignment of base stations in cellular phone networks. At first sight, one might think that the conflict graph is planar in this problem, and the Four Color Theorem can be used, but it is not true: if there are lots of stations in small region, then they are all close to each other, therefore they form a large clique in the conflict graph. Instead, the conflict graph is a unit disk graph, where each vertex corresponds to a disk in the plane with unit diameter, and two vertices are connected if and only if the corresponding disks intersect. A 3-approximation algorithm for coloring unit disk graphs is given in [9], yielding a 3-approximation for the frequency assignment problem.

G. Final Exam Timetabling as a Grouping Problem

Linear Linkage Encoding (LLE) is a recently proposed representation scheme for evolutionary algorithms. This representation has been used only in data clustering. However, it is also suitable for grouping problems. Final exam timetabling requires satisfactory assignment of timetable slots (periods) to a set of exams. Each exam is taken by a number of students, based on a set of constraints. In most of the studies, NE like representations are used. In [10], a randomly selected light or a heavy mutation followed by a hill climbing method was applied. Various combinations of constraint satisfaction techniques with genetic algorithms can be found in [12]. Paquete et. al. [14] applied a multi-objective evolutionary algorithm based on pareto ranking with two objectives: minimize the number of conflicts within the same group and between groups. Wong et. al [15] applied a GA with a non-elitist replacement strategy. After genetic operators are applied, violations are repaired with a hill climbing fixing process. In their experiments a single problem instance was used. Ozcan et. al. [13] proposed a memetic algorithm (MA) for solving final exam timetabling at Yeditepe University. MA utilizes a violation directed adaptive hill climber. Considering the task of minimizing the number of exam periods and removing the clashes, final exam timetabling reduces to the graph coloring problem [11].

H. Map Coloring and GSM Mobile Phone Networks

Groups Special Mobile (GSM) is a mobile phone network where the geographical area of this network is divided into hexagonal regions or cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the neighbors. Since GSM operate only in four different frequency ranges, it is clear by the concept of graph theory that only four colors can be used to color the cellular regions. These four different colors are used for proper coloring of the regions. Therefore, the vertex coloring algorithm may be used to assign at most four different frequencies for any GSM mobile phone network. Given a map drawn on the plane or on the surface of a sphere, the four color theorem assets that it is always possible to color the regions of a map properly using at most four distinct colors such that no two adjacent regions are assigned the same color. Now, a dual graph is constructed by putting a vertex inside each region of the map and connect two distinct vertices by an edge iff their respective regions share a whole segment of their boundaries in common. Then proper coloring of the dual graph gives proper coloring of the original map. Since, coloring the regions of a planar graph G is equivalent to coloring the vertices of its dual graph and vice versa.[16] By coloring the map regions using four color theorem, the four frequencies can be assigned to the regions accordingly.[17]

Graph coloring is the assignment of labels or colors to elements of a graph (vertices or edges) subject to certain constraints. In this project, we will consider how as few colors as possible can be assigned to vertices of a graph so that no two neighboring vertices (vertices connected by an edge) have the same color. The convention of using colors comes from coloring countries on a map where each country should have a different color from its neighbour. However, countries on a map is an example of a planar graph and for planar graphs, four colours are enough. In the case of non-planar graphs, we do not know how many colors are required.

In graph theory, a planar graph is a graph that can be embedded in the plane, i.e, it can be drawn on the plane in such a way that its edges intersect only at their endpoints. An example of a map coloring (planar case) is shown in Figure 8 where neighboring states are colored using different colors.

Given a map drawn on the plane or the surface of a sphere, the famous four colour theorem asserts that it is always possible to properly color the regions of the map such that no two adjacent regions are assigned the same color, using at most four distinct colors[18][20][21].

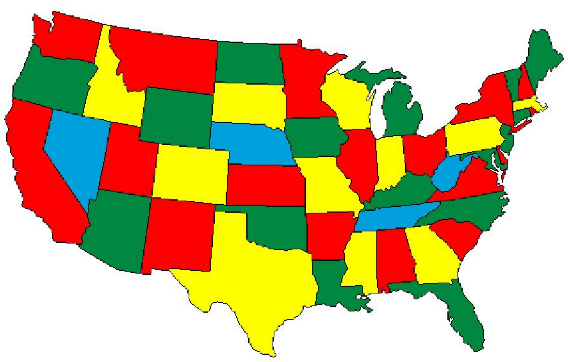

Fig. 8. Map coloring of the states in the U.S.

For any given map, we can construct its dual graph as follows. Put a vertex inside each region of the map and connect two distinct vertices by an edge if and only if their respective regions share a whole segment of their boundaries in common. Then, a proper vertex coloring of the dual graph yields a proper coloring of the regions of the original map.

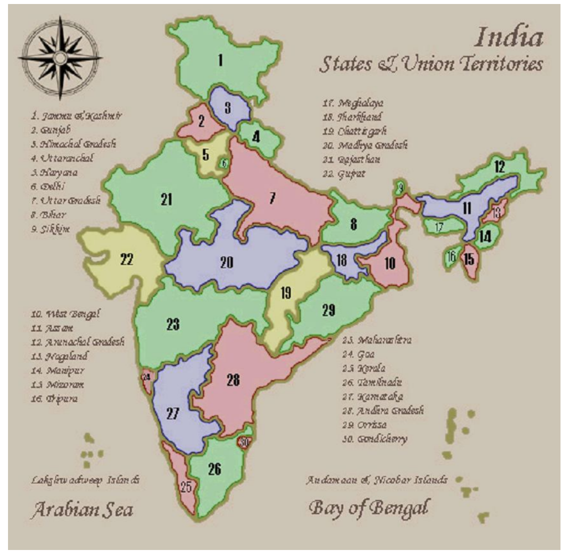

Fig. 9. The map of India

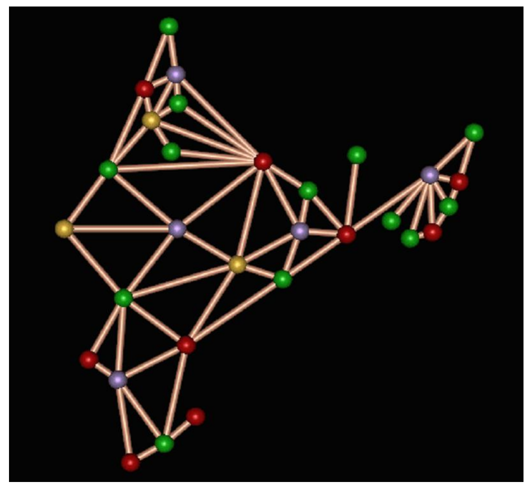

Fig. 6. The dual graph of the map of India

We use the vertex coloring algorithm **[19]** to find a proper coloring of the map of India with four colors, see figures 5 and 6 above. The Groupe Spécial Mobile (GSM) was created in 1982 to provide a standard for a mobile telephone system. The first GSM network was launched in 1991 by Radiolinja in Finland with joint technical infrastructure maintenance from Ericsson. Today, GSM is the most popular standard for mobile phones in the world, used by over 2 billion people across more than 212 countries. GSM is a cellular network with its entire geographical range divided into hexagonal cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the immediate vicinity. GSM networks operate in only four different frequency ranges. The reason why only four different frequencies suffice is clear: the map of the cellular regions can be properly colored by using only four different colors! So, the vertex coloring algorithm may be used for assigning at most four different frequencies for any GSM mobile phone network, see figure 10 below.

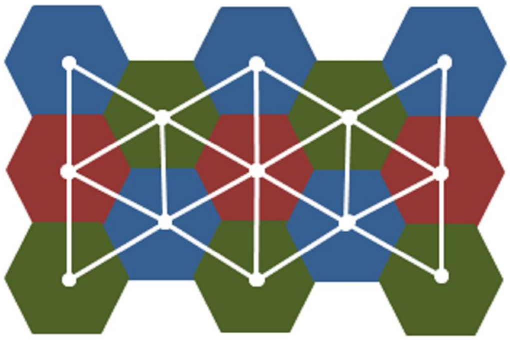

Fig. 10. The cells of a GSM mobile phone network

I. Student Time Table

Timetabling is the allocation, subject to constraints, of given resources to objects in space-time domain to satisfy a set of desirable objectives as nearly as possible. Particularly, the university timetabling problem for courses can be viewed as fixing in time and space a sequence of meetings between instructors and students, while simultaneously satisfying a number of various essential conditions or constraints.

Graph Coloring Algorithm was used to generate the student weekly time table in a typical university department. The problem was a Node-Point problem and it could not be solved in the polynomial domain. Various constraints in weekly scheduling such as lecturer demands, course hours and laboratory allocations were confronted and weekly time tables were generated for 1st, 2nd, 3rd and 4th year students in a typical semester.

In a typical semester, the courses are required to be scheduled at different times in order to avoid conflict. The problem of determining the minimum number (or a reasonable number) of time slots needed to schedule all the courses subject to restrictions is a graph coloring problem. **Figure 11** illustrates a simple timetabling problem instance in which we have five courses to be scheduled: Physics, Calculus, Electronics, Microprocessors, and Operating Systems.

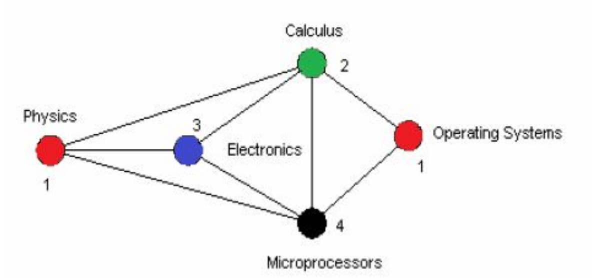

Fig 11. Colored graph G for 5 courses.

In Ta**ble 1** below, an asterisk indicates those pairs of courses that would cause a timetabling conflict if both were scheduled at the same time.

	Physics	Calculus	Electronics	Microprocessors	Operating Systems
Physics		*	*	*	
Calculus	*		*	*	*
Electronics	*	*		*	
Microprocessors	*	*	*		*
Operating Systems		*		*	

Table 1: Course distribution

The cause for potential conflict could be any of the following example restrictions:

a) Courses Calculus and Electronics might be taught by the same professor,

b) Courses Microprocessors and Operating Systems might be taken by the same student.

Given the list of courses Physics, Calculus, Electro- nics, Microprocessors, and Operating Systems along with the set of potential conflicts, we can create a conflict-free timetable of courses by transforming the **Table 1** to the corresponding conflict graph G in **Figure 11**, and finding a minimum coloring. A vertex in G represents a course, an edge represents a pair of courses that conflict, and a color represents the period in which that particular course is to be scheduled. We see that four periods are required to schedule all the courses without conflict: (G) = 4. According to the coloring, we can schedule courses Physics and Operating Systems for Period 1, and courses Calculus, Electronics, and Microprocessors for Periods 2, 3, and 4 respectively. With course timetabling, it is often desirable that courses do not "student-conflict" (i.e., those two courses sharing a common student will not be scheduled at the same time).

III. Conclusion

An overview is presented especially to project the idea of graph coloring. Researches may get some information related to graph coloring and its applications in computer field and can get some ideas related to their field of research. This paper gives an overview of the applications of graph coloring in heterogeneous fields to some extent but mainly focuses on the computer science applications that uses graph coloring concepts. Various papers based on graph coloring have been studied related to scheduling concepts, computer science applications and an overview has been presented here.

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